AD-A040 077 VIRGINIA POLYTECHNIC INST AND STATE UNIV BLACKSBURG --ETC F/G 11/4
RESIDUAL STRENGTH DEGRADATION MODEL AND THEORY OF PERIODIC PROO--ETC(U)
NOV 76 J N YANG, M D LIU F33615-75-C-5112

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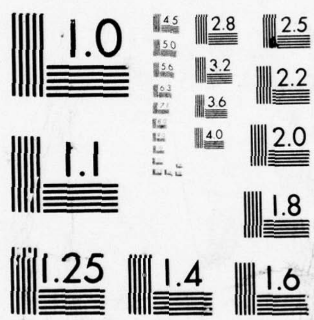

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RESIDUAL STRENGTH DEGRADATION MODEL AND THEORY OF PERIODIC PROOF TESTS FOR GRAPHITE/EPOXY LAMINATES

VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
BLACKSBURG, VA 24061

NOVEMBER 15, 1976

FINAL REPORT JANUARY 1976 - NOVEMBER 1976

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FOR THE DIRECTOR

S. W. Tsai

S. W. TSAI, Project Monitor
Chief, Mechanics & Surface Interactions Branch
Nonmetallic Materials Branch

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AIR FORCE - 13 APRIL 77 - 200

(19) REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER (18) AFML-TR-76-225

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle) (6) Residual Strength Degradation Model and Theory of Periodic Proof Tests for Graphite/Epoxy Laminates.

5. TYPE OF REPORT & PERIOD COVERED Final Report Jan.1, 1976-Nov.15, 1976

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s) (10) Jann N. Yang, M. D. Liu

8. CONTRACT OR GRANT NUMBER(s) (15) F33615-75-C-5112 new

9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Engineering Science & Mech. Virginia Poly. Institute & State Univ. Blacksburg, VA 24061 (16)

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 734003A3 (17) 03

11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Materials Laboratory (AFML/MBM) Air Force Wright Aeronautical Laboratories Wright-Patterson AFB, OH 45433

12. REPORT DATE (11) 15 November 1976

13. NUMBER OF PAGES 50 (12) 57p.

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Same

(9) Final rept. 1 Jan-15 Nov 76

15. SECURITY CLASS. (of this report) Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

62102F

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Fatigue of Composites
Residual Strength Degradation
Statistical Strength Distribution
Graphite/Epoxy Composites
Periodic Proof Loads
Composite Reliability

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A fatigue residual strength degradation model has been derived based on the assumption that the residual strength decreases monotonically. The model is verified experimentally by use of Gr/E unnotched laminates under constant amplitude fatigue. The theory of periodic proof tests and reliability prediction for composites have been established. Experimental data for specimens under periodic proof tests and fatigue loading have been generated. It is shown that the correlation between the →

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experimental results and the theory of periodic proof tests is outstanding.

FOREWORD

This report represents one part of the research results sponsored by the Air Force Materials Laboratory under contract No. F33615-75-C-5112 to Virginia Polytechnic Institute and State University. The research was performed by Dr. J. N. Yang and Mr. M. D. Liu. The analytical work, analyses of test data and the preparation of this report were conducted during the period September 1 to November 15, 1976, at the George Washington University, Department of Civil, Mechanical and Environmental Engineering with which Dr. J. N. Yang is presently associated. The project engineer for the Air Force Materials Laboratory was Dr. S. W. Tsai, Chief, Mechanics and Surface Interaction Branch.

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INTRODUCTION

In Ref. 1, a preliminary investigation has been made for the reliability prediction of composites under periodic proof tests in service, along with the determination of the optimal periodic proof test. The theory of periodic proof tests and the reliability prediction for composites has been based on a particular residual strength degradation model (Refs. 1-4). It is the purpose of this paper to verify experimentally the theory of proof tests established in Ref. 1 and the validity of the residual strength degradation model used.

Because of the importance of the residual strength degradation model to the reliability prediction of composites, a new theoretical derivation of such a model, based on the assumption that the residual strength degrades monotonically, is presented. The experimental verification of such a model using constant amplitude fatigue data is then followed in Section II.

After the experimental verification of the residual strength degradation model, two series of experimental data under periodic proof tests are presented in Section III. The theoretical prediction of the residual strength degradation under both cyclic fatigue loading and periodic proof tests is derived and correlated with experimental data.

It is shown that (i) the theoretical residual strength degradation model correlates reasonably well with the experimental data for Gr/E unnotched laminates under constant amplitude fatigue loading,

(ii) the theory of periodic proof tests established in Ref. 1 correlates extremely well with the experimental data, (iii) damage to Gr/E unnotched laminates, in term of static strength and fatigue strength, due to repetitive proof tests appears to be negligible for the proof load level up to 90% of the mean ultimate strength, and (iv) the residual strength degradation model, the theory of periodic proof tests, and the methodology for reliability prediction of composites under periodic proof tests in service established in Ref. 1 can be used with confidence for Gr/E unnotched laminates under constant amplitude fatigue loads.

The importance of proof testing as a means to ensure the reliability of composites and its performance over the entire anticipated life has been emphasized in the literature (Refs. 1-4). The present effort along with that of Ref. 1 will provide rational basis for the designer to determine the optimal proof load level as well as the optimal numer of periodic proof tests in service, in order to ensure a prescribed level of composite reliability. It is an integral part of life assurance program for composites.

SECTION II

RESIDUAL STRENGTH DEGRADATION MODEL

(1) Theoretical Derivation

Experimental fatigue data (Refs. 2-3) on unnotched composite laminates indicate that the residual strength $R(n)$ after n fatigue cycles is a monotonically decreasing function of n. Assuming that the slope of the residual strength $R(n)$ is inversely proportional to some power c-1 of the residual strength $R(n)$ itself, one obtains a rate equation,

$$dR(n)/dn = -f(S,\omega,\varepsilon)/cR^{c-1}(n) \quad (1)$$

in which $f(S,\omega,\varepsilon)$ is a function of the maximum cyclic stress S, the loading frequency ω, and the stress ratio ε. In Eq. 1, c is a constant. For the sake of simplicity, the loading frequency ω and the stress ratio ε will be fixed so that

$$f(S,\omega,\varepsilon) = f(S) \quad (2)$$

Integration of Eq. 1 from n_0 to n_1 cycles leads to the following result,

$$R^c(n_1) = R^c(n_0) - f(S)(n_1-n_0) \quad (3)$$

in which $f(s)$ is a function of maximum stress S.

With $n_0 = 0$, $n_1 = n$, Eq. 3 reduces to

$$R^c(n) = R^c(0) - f(S)n \quad (4)$$

Eq. 4 expresses the residual strength at n cycles, $R(n)$, in terms of the ultimate strength, $R(0)$, the number of load cycle, n, and the maximum stress, S. The shape of strength degradation given in Eq. 4 depends on the value of c (Refs. 1,2).

It is reasonable (Refs. 1-7) to assume that the statistical distribution of the ultimate strength, $R(0)$, follows a two-parameter Weibull distribution,

$$F_{R(0)}(x) = P\left[R(0) \leq x\right] = 1 - \exp\left[-(x/\beta)^{\alpha}\right] \tag{5}$$

in which α is the shape parameter and β is the scale parameter or the characteristic strength.

It is assumed that fatigue failure occurs as soon as the residual strength $R(n)$ is reduced to the value of maximum stress level S. Let N be the number of cycles at which fatigue failure occurs under the maximum cyclic stress S. Then, at the moment of fatigue fracture, $R(n) = S$ and $n = N$. The number of cycles to fatigue failure N can be obtained from Eq. 4 by imposing the condition of fracture, i.e., $R(n) = S$, $n = N$, as follows; $S^c = R^c(0) - f(S)N$. Hence, one obtains

$$N = \left[R^c(0) - S^c\right] / f(S) \tag{6}$$

Since the ultimate strength, $R(0)$, is a statistical variable, it follows from Eq. 6 that the number of cycles to fatigue failure, N, is also a statistical variable. The statistical distribution of fatigue life N can be obtained from the statistical distribution

of R(0) given by Eq. 5 through the transformation of Eq. 6 as follows;

$$F_N(n) = P\left[N \leq n\right] = P\left[\left\{R^c(0) - S^c\right\}/\ f(s) \leq n\right]$$
$$= P\left[R(0) \leq \left\{nf(s) + S^c\right\}^{1/c}\right] \tag{7}$$

Substitution of Eq. 5 into Eq. 7 yields

$$F_N(n) = 1 - \exp\left\{-\left[\frac{n + \left[S^c/f(S)\right]}{\beta^c/f(S)}\right]^{\alpha/c}\right\} \tag{8}$$

It is observed from Eq. 8 that the statistical distribution of fatigue life N (number of cycles to fatigue failure) follows a three parameter Weibull distribution with a lower bound at $N_0 = -S^c/f(S)$.

For the sake of clarity, two different situations will be considered separately in the following, although they lead to identical residual strength degradation model.

(i) Low stress amplitude fatigue

In general, when we are interested in the region where the maximum stress amplitude S is small such that $(S/\beta)^c << 1$, i.e., the number of cycles to failure is high, Eq. 8 can be approximated in the following;

$$F_N(n) \simeq 1 - \exp\left\{-\left[\frac{n}{\beta^c/f(s)}\right]^{\alpha/c}\right\} \tag{9}$$

indicating that if the ratio of the stress level S to the characteristic ultimate strength β is such that $(S/\beta)^c << 1$, the statistical distribution of fatigue life N is approximately a two-parameter

Weibull distribution with the shape parameter α/c and the scale parameter (characteristic life) $\beta^c/f(S)$.

Supposing that the constant amplitude fatigue data is available, one obtains a classical S-N* curve by connecting the characteristic lives associated with each stress level S. The S-N* curve can be expressed classically as

$$KS^bN^* = 1 \tag{10}$$

in which K and b are constants and N* is the characteristic life associated with a maximum stress level S. Eq. 10 can be written as

$$N^* = (KS^b)^{-1} \tag{11}$$

Comparison of Eqs. 9 and 11 leads to the result, $\beta^c/f(s) = (KS^b)^{-1}$. Hence, the functional form of f(S) is obtained as

$$f(S) = \beta^c KS^b \tag{12}$$

Substituting Eq. 12 into Eq. 4, one obtains

$$R^c(n) = R^c(0) - \beta^c KS^b n \tag{13}$$

in which c, b and K are constants to be determined from experimental data. Eq. 13 is the equation describing the residual strength degradation under constant amplitude cyclic loads.

If Eq. 12 is substituted into Eq. 3, one obtains a general equation

$$R^c(n_1) = R^c(n_0) - \beta^c K S^b (n_1 - n_0) \tag{14}$$

It is mentioned that the model presented herein, Eqs. 13, and 14, is similar to the one proposed by Halpin and Waddoups (Refs. 3-4). However, they derived the strength degradation based on the assumption of crack propagation. The model derived herein is based on the assumption that the residual strength is a monotonically decreasing function of load cycles n.

(ii) High stress amplitude fatigue

For composite laminates where the endurance limit is rather high, fatigue failure will not occur unless the maximum stress S is very high. Under this circumstance, $(S/\beta)^c$ may not be negligible, although c is usually large, e.g., c = 10 for Gr/E composites.

Letting

$$\gamma = (S/\beta)^c ; \; N_1 = \beta^c/f(s) \tag{15}$$

and introducing a new random variable $\tilde{N}$, indicating a shifted fatigue life by an amount of γN_1 to the right, i.e.,

$$\tilde{N} = N + \gamma N_1 \tag{16}$$

one obtains from Eq. 8 the statistical distribution of the shifted fatigue life $\tilde{N}$ as follows;

$$F_{\tilde{N}}(n) = P\left[\tilde{N} \leq n\right] = 1 - \exp\left\{ - (n/N_1)^{\alpha/c} \right\} \tag{17}$$

It follows from Eq. 17 that the statistical distribution of the shifted fatigue life $\tilde{N}$ is a two parameter Weibull distribution with a shape parameter α/c and a characteristic life of N_1.

Supposing that the fatigue data is available for $\tilde{N}$, one obtains a classical S-N_1, curve by connecting the characterstic lives associated with each stress level S. Such a S-N_1 curve can be approximated by

$$KS^bN_1 = 1 \tag{18}$$

in which K and b are constants which are different from K and b in Eqs. 10-14.

Substitution of N_1 appearing in Eq. 18 into Eq. 15 leads to the functional form for f(S),

$$f(S) = K\beta^c S^b \tag{19}$$

Substituting Eq. 19 into Eq. 4, one obtains

$$R^c(n) = R^c(0) - K\beta^c S^b n \tag{20}$$

Eq. 20 is identical to Eq. 13 except that the meaning of the constants K and b are different. This indicates that the form of the fatigue residual strength degradation model of Eq. 20 can be used for both the low stress amplitude and the high stress amplitude fatigue.

It is very important to emphasize that in the process of deriving the residual strength degradation model, Eq. 20 or 13, parameters, c, b and K are assumed to be determined from the stress-characteristic life S-N curve of statistical fatigue data (see Eqs. 10 and 18). The generation of statistical data for Eqs. 10 and 18 is extremely tedious and should not be attempted. It only serves as a vehicle for us to derive the residual strength degrada-

tion model only. More efficient analysis technique and test procedure will be presented in the following, such that the number of test data required for the determination of parameter values c, b and K can be minimized.

Once the parameter values K, b and c have been determined from experimental data, Eq. 20 can be used easily to find the statistical distribution of the residual strength, $R(n)$, after n cycles, and that of the fatigue life N for any value of maximum cyclic stress S.

The statistical distribution of the residual strength $R(n)$ after n fatigue cycles can be obtained from the statistical distribution of $R(0)$ given by Eq. 5 through the transformation of Eq. 20 as follows;

$$\begin{aligned} F_{R(n)}(x) &= P\left[R(n) \le x\right] = P\left[R^c(0) - K\beta^c S^b n \le x^c\right] \\ &= P\left[R(0) \le (x^c + K\beta^c S^b n)^{1/c}\right] \\ &= 1 - \exp\left\{-\left[(x^c + K\beta^c S^b n)/\beta^c\right]^{\alpha/c}\right\} \end{aligned} \tag{21}$$

Eq. 21 indicates that the statistical distribution of the residual strength $R_n(x)$ after n stress cycles is a three parameter Weibull distribution with a lower bound at $-(K\beta^c S^b n)^{1/c}$. This comes from the fact that some of the specimens will not survive up to n stress cycles. Hence, the distribution function should be written as

$$\begin{aligned} F_{R(n)}(x) &= 0 & ; x < 0 \\ &= 1 - \exp\left\{-\left[(x^c + K\beta^c S^b n)/\beta^c\right]^{\alpha/c}\right\} & ; x \ge 0 \end{aligned} \tag{22}$$

in which $F_{R(n)}(S) = 1 - \exp\left\{ - \left[(S^c + K\beta^c S^b n)/\beta^c \right]^{\alpha/c} \right\}$ is the probability that the specimen will fracture before n cycles, i.e., the probability that the specimen will not survive n stress cycles. Note that there is a discontinuity in the distribution function, $F_{R(n)}(x)$, at x = 0, and hence there is a dirac delta at x = 0 for the probability density function.

The number of cycles N to fatigue failure can be obtained from Eq. 20 by imposing the condition of fracture, i.e., n = N, R(n) = S, as follows;

$$N = \left[R^c(0) - S^c \right] / K\beta^c S^b \tag{23}$$

The statistical distribution of fatigue life N can be obtained from that of the static strength R(0) given by Eq. 5 through the transformation of Eq. 23 as follows:

$$\begin{aligned} F_N(n) &= P\left[N \le n \right] = P\left[R(0) \le (nK\beta^c S^b + S^c)^{1/c} \right] \\ &= 1 - \exp\left\{ - \left[(nK\beta^c S^b + S^c)/\beta^c \right]^{\alpha/c} \right\} \\ &= 1 - \exp\left\{ - \left[\frac{n + (S^c/K\beta^c S^b)}{1/KS^b} \right]^{\alpha/c} \right\} \end{aligned} \tag{24a}$$

Eq. 24a indicates that the fatigue life distribution is a three parameter Weibull distribution with a lower bound at $-S^c/\beta^c KS^b$. This comes from the fact that some of the specimens will fail within one cycle, when their ultimate strengths are smaller than S. As a result, it should be written as

$$F_N(n) = 0 \qquad ; n < 1$$

$$= 1 - \exp\left\{- \left[\frac{n + (S^c/K\beta^c S^b)}{1/KS^b}\right]^{\alpha/c}\right\}; \; n \geq 1 \tag{24b}$$

where $F_N(1)$ is the probability of failure in one cycle, i.e., the probability that the ultimate strength is smaller than the maximum applied stress S. Again, there is a discontinuity in the distribution function $F_N(n)$ at $n = 1$, and hence a dirac delta at $n = 1$ in the probability density function.

(2) Test Plan and Analysis Technique for the Determination of Parameter Values.

Economical test plan and efficient analysis technique are presented herein to minimize the number of test data required for the determination of parameter values c, b and K appearing in the residual strength *degradation* model. Let $x_1, x_2, \ldots, x_m$ be static ultimate strengths of a set of m specimens, referred to as static specimens. The first three central moments of the static strength data are given as follows;

$$m_1 = \mu = \frac{1}{m} \sum_{i=1}^{m} x_i$$

$$m_2 = \frac{1}{m} \sum_{i=1}^{m} (x_i - m_1)^2$$

$$m_3 = \frac{1}{m} \sum_{i=1}^{m} (x_i - m_1)^3 \tag{25}$$

Note that m_1 is the sample mean, $\sqrt{m_2}$ is the sample standard deviation and $m_3^{1/3}/ m_1$ is the sample skewness

A set of J specimens are subjected to constant amplitude fatigue at various maximum stress level S, referred to as fatigue specimens. Some specimens will fracture under fatigue within 10^6 or 10^7 cycles, and some don't. When the specimen does not fail, the fatigue test is stopped and its residual strength is measured. Furthermore, the residual strength of the specimen at the instant of fracture in fatigue is equal to the maximum cyclic stress S. As a result, a set of J data points of residual strengths, denoted by (R_1, S_1, n_1), (R_2, S_2, n_2), ..., (R_J, S_J, n_J), is obtained from fatigue specimens. Note that the ith data point (R_i, S_i, n_i), indicates that the ith specimen is subjected to a maximum cyclic stress S_i for n_i cycles and its residual strength is R_i. If the specimen fracture at n_i cycles, then $R_i = S_i$.

Now the residual strength $R_i (i = 1,2,...,J)$ of the fatigue specimens can be converted into (or recovered to) the equivalent ultimate strength, denoted by $R_i(0)$, through the strength degradation model of Eq. 20, i.e.,

$$R_i(0) = \left[R_i^c + K\beta^c S_i^b n_i\right]^{1/c} \quad ; \quad i = 1,2,..., J. \tag{26}$$

in which $R_i(0)$ would have been the static ultimate strength if the ith fatigue specimen were not subjected to fatigue.

Using Eq. 26, one obtains from fatigue data a set of J equivalent ultimated static strength data, i.e., $R_1(0)$, $R_2(0)$..., $R_J(0)$. The first three central moments, denoted by μ_1, μ_2 and μ_3 are as follows;

$$\mu_1 = \frac{1}{J} \sum_{i=1}^{J} R_i(0)$$

$$\mu_2 = \frac{1}{J} \sum_{i=1}^{J} \left[R_i(0) - \mu_1 \right]^2$$

$$\mu_3 = \frac{1}{J} \sum_{i=1}^{J} \left[R_i(0) - \mu_1 \right]^3 \tag{27}$$

Theoretically, if the number m of static ultimate strength data and the number J of fatigue data approach infinity, the statistical distribution of $x_i (i=1,2,...,m)$ should be identical to the statistical distribution of $R_i(0)$ $(i=1,2,...,J)$, and hence $m_1 = \mu_1$, $m_2 = \mu_2$ and $m_3 = \mu_3$, if the residual strength degradation model (Eq. 20) is correct. In reality, however, since the number of data points is finite, sampling fluctuation can not be avoided, and hence it may not be possible to match the first three central moments to determine c, b and K. Consequently, c, b and K are obtained by minimizing the mean square difference Δ between the first three central moments i.e.,

$$\Delta = (m_1 - \mu_1)^2 + g_1 \left(\sqrt{m_2} - \sqrt{\mu_2} \right)^2 + g_3 \left(\sqrt[3]{m_3} - \sqrt[3]{\mu_3} \right)^2 \tag{28}$$

where g_1 and g_2 are assigned weighting positive values to indicate the relative importance of matching the mean, the standard deviation and the skewness.

It becomes clear now that in order to determine the parameter values of the residual strength degradation model (Eq. 20), i.e., K, b and c, one needs to test a set of m static data and a set of J fatigue data. Since the statistical dispersion of static strength data is small, m is not necessarily large. The total

number of fatigue data J depends on the ranges of fatigue life and stress levels which are of interest to us. Experience indicates that $J \simeq 30$ is sufficient for the stress range $S \gtrsim 60\%$ of the mean ultimate strength.

(3) Experimental Verification

A 2' x 2' panel of graphite/epoxy laminate was curve using the Gr/E tape purchased from Hercules AS3501. This panel, referred to as panel 3, was cut into approximately 70 specimens. The layup of the laminates is $(0,90,\pm 45)_s$ and the size of the specimen is shown in Fig. 1.

Twelve (12) specimens are tested statically and their static ultimate strengths are shown in Table I. The two-parameter Weibull distribution (Eq. 5) is used to fit the data as shown by the solid curve of Fig. 2(a). A shape parameter $\alpha = 14.778$ and a characteristic ultimate strength $\beta = 78.109$ ksi have been obtained.

Twenty one (21) specimens are subjected to fatigue cycles at various maximum stress levels S and the results are tabulated in Table II. One specimen, No. 2-12, did not fail at one million cycles and its residual strength was measured. In addition, 4 specimens are fatigued at a maximum stress level 72.5% of the mean ultimate strength $\mu = 75.391$ ksi up to 30,000 cycles. The fatigue test is then stopped and their residual strengths are measured. Results are shown at the bottom of Table II. All the fatigue tests are performed at a frequency of 20Hz with a

stress ratio 0.1 on an Instron cyclers. Both the static data and the fatigue data (Table I and II) are plotted in Fig. 3.

A total of 12 static data points in Table I and 25 fatigue data points in Table II are used to determine the parameter values c, b and K using the method of analysis described in Eq. 25-28; with the result,

$$C = 10.818;\ b = 17.78;\ K = 1.8285 \times 10^{-36} \qquad (29)$$

As a first indication of the goodness of the residual strength degradation model (Eq. 20), a set of 25 fatigue data (Table II) is converted into a set of equivalent static ultimate strength data $R_i(0)$ $(i = 1,2,\ldots,25)$ through Eq. 26 using c, b and K values given by Eq. 29. Then, this set of 25 equivalent ultimate strength data is normalized with respect to β and is plotted in Fig. 2(b) as circles where the solid curve is the Weibull distribution of 12 static strength data (Table I).

It can be observed from Fig. 2(b) that the correlation between the fatigue converted ultimate strength data (circles) and the static ultimate strength distribution is outstanding. This is a necessary condition for the residual strength degradation model, Eq. 20, to be reasonable.

Since R(0) and R(n) are random variables, Eq. 20 can also be written as

$$R_p^c(n) = R_p^c(0) - K\beta^c S^b n \qquad (30)$$

in which $R_p(n)$ and $R_p(0)$ represent, respectively, the p reliability point. A plot of Eq. 30 is given by Fig. 4 along with 25 fatigue data (Table II).

With the parameter values of c, b and K as well as Eq. 20, it is possible to predict the residual strength distribution under any maximum stress level S at any number of cycles n by use of Eq. 22.

To verify the validity of the residual strength degradation model (Eq. 20), a set of 8 specimens are fatigue tested at the maximum stress level of $S = 0.7\mu = 52.77$ksi up to 26,000 cycles. One specimen fails at 17,540 cycles. The residual strengths of the remaining 7 surviving specimens are given in Table III.

According to the theoretical model, Eq. 22, the probability of failure before 26,000 cycles is $F_{R(n)}(S) = 0.1135$ (see Eq. 22). The experimental data shows one failure out of 8 specimens, i.e., $1/(8 + 1) = 0.11$, indicating that the theoretical prediction is very good. The theoretical prediction (Eq. 22) is plotted in Fig. 5(a) as a solid curve and the experimental data as circles. Considering the sampling fluctuation due to small sample size, the correlation between test data and theoretical prediction appears to be reasonable.

Furthermore the conditional distribution function of the residual strength $R^*(n)$ for surviving specimens can also be predicted theoretically using Eq. 22 as follows;

$$
\begin{aligned}
F_{R*(n)}(x) &= 1 - P\left[R(n) > x \mid R(\iota) > S\right] \\
&= 1 - \left\{P\left[R(n) > x\right] / P\left[R(n) > S\right]\right\} \\
&= 1 - \exp\left\{F_{R(n)}(S) - \left[\frac{x^c + K\beta^c S^b n}{\beta^c}\right]^{\alpha/c}\right\} ; \; x > S
\end{aligned}
\tag{30}
$$

in which $F_{R(n)}(S)$ is given by Eq. 22.

The theoretical prediction (Eq. 30) is plotted in Fig. 5(b) as solid curve and the experimental data as circles. Again, the correlation is reasonable in view of small sample size.

SECTION III

PERIODIC PROOF TESTS

(1) Experimental Data

A set of 14 specimens is subjected to a proof load $r_0 = 0.9\mu = 67.816$ ksi prior to fatigue. After the specimen passes the initial proof load r_0, it is subjected to 26,000 fatigue cycles at the maximum stress level $S = 0.7\mu \approx 52.78$ ksi. A second proof load r_0 is further applied to the specimen and then followed by another 26,000 fatigue cycles. The residual strengths of surviving specimens are measured at this point. A loading history is schematically shown in Fig. 6. Results of residual strengths are shown in Table IV.

Note that there is a possibility that the specimen may fail under the initial proof load or the second proof load. Test results show that 3 specimens fail under the 2nd proof test and their residual strengths are indicated by a star in Table IV. No

specimen fails under the initial proof test or by fatigue.

A second set of 11 specimens is subjected to the same loading history as shown in Fig. 6, except that the proof load level is 85% of the mean static ultimate strength, i.e., $r_0 = 0.85\mu = 64.1$ ksi. Residual strengths of 8 surviving specimens are given in Table V. Three specimens fail under the second proof test and their residual strengths are indicated in Table V by a star. No specimen fails under the initial proof test or by fatigue.

(2) Theoretical and Experimental Correlation

Theoretical derivation for the statistical distribution of the residual strength of surviving specimens under periodic proof tests and service load has been presented in Ref. 1. The results for the particular case of constant amplitude fatigue is presented in the Appendix. Theoretical predictions for testing conditions associated with two sets of experimental data are presented in Fig. 7 and 8 as solid curves, along with the experimental data indicated by circles.

It can be observed from Figs. 7 and 8 that the correlation between the theory of periodic proof tests and the experimental results is surprisingly good for both cases.

According to the theoretical predictions derived in the appendix, the following conclusions are made for the first set of experimental data (Table IV).

(i) If the specimen passes a proof load $r_0 = 0.9\mu = 67.852$ ksi, it will survive at least 26,750 fatigue cycles at the maximum stress level $S = 0.7\mu = 52.7$ ksi. This can by computed easily from Eq. 23 by setting $R(0) = r_0$. This conclusion is confirmed by the experimental data that none of the 14 specimens fails in fatigue, i.e., when the specimen passes either the initial proof test or the second proof test, it does not fail in the subsequent 26,000 cycles of fatigue load. This is an essential and important demonstration that the periodic proof tests not only sensors unnotched specimens that are weak in static strength, but also screens out specimens having weak fatigue strength, thus guarranteeing a minimum fatigue life for specimens when they pass certain level of proof load. This is an essential advantage for the application of periodic proof tests as advocated in Refs. 1-4.

(ii) The probability of failure of a specimen under the initial proof load $r_0 = 0.9\mu$ is computed from Eq. A - 4 in the Appendix as 0.117. The probability of failure of a specimen under the second proof test is computed from Eq. A - 7 in the Appendix as 0.161. As a result, the total probability that a specimen will fail under two proof tests is 0.278. Consequently, for the 14 specimens tested, an average of $14 \times 0.278 = 3.9$ specimens will be destroyed by proof tests. The experimental results show 3 specimens failed under the second proof test. Considering the sampling fluctuating associated with small sample size, the correlation between the theory and the experimental data is reasonable.

(iii) The statistical distribution of the residual strength of the specimens surviving 2 proof tests and two service intervals, where each interval is 26,000 fatigue cycles at $S = 0.7\mu$, has been derived in Eq. A-6 of the Appendix. This theoretical distribution is plotted in Fig. 7 as solid curve. Also plotted in Fig. 7 as circles are the residual strengths of the experimental data (Table V). It can be observed that the correlation between the theoretical prediction and experimental results is excellent.

(iiii) The fact that good correlation exists between the theory of periodic proof tests and experimental data indicates that both the residual strength degradation model and the theory of periodic proof tests are reasonable and can be used with certain degree of confidence. No good correlation can be obtained, if either the residual strength degradation model or the theory of periodic proof tests is unreasonable.

The conclusion for the second set of experimental data are given in the following:

(i) According to Eq. 23, the specimen will survive at least 14,000 fatigue cycles at $S = 0.7\mu = 52.78$ ksi after passing a proof load $r_0 = 0.85\mu = 64.1$ ksi. Since the specimens are subjected to two service intervals each with 26,000 fatigue cycles, there is a slight probability that the specimen may fail by fatigue. The probability of failure under initial proof test is computed from Eq. A - 4 as 0.0525. The probabilities of failure (a) in the first service interval, (b) under the second proof

test and (c) in the second service interval, are computed, respectively, as $P_1 = 0.064$, $B_1 = 0.078$ and $P_2 = 0.073$ (see Eqs. A-11 to A-13 of the Appendix). As a result, the total probability that the specimen will not survive up to the end of the second service interval is 0.2675. Hence, the average number of specimens expected to fail either in fatigue or under proof tests is $0.2675 \times 11 = 2.94$ specimens. The experimental results show (Table III) that 3 specimens fail before the end of the second service interval. The correlation between the theory and experimental results is good.

(ii) The theoretical prediction for the residual strength distribution of surviving specimens has been derived in Eq. A - 17 of the Appendix, and is plotted as solid curve in Fig. 8. Also plotted in Fig. 8 as circles are the results of experimental data. Again, the correlation between the theory of periodic proof tests and experimental data is excellent.

(3) Damage Due to Proof Tests

It has been a concern that proof tests may damage composites in terms of the reduction of both the static strength and the fatigue strength as discussed in Refs.1-4. Damage in static strength may come from the fact that under one cycle of large proof load, the existing crack may increase its size thus reducing the static strength. It has been shown experimentally that such a damage is negligible for Gr/E $\pi/4$ laminates (Ref. 9).

Damage in fatigue strength may come from the initiation of cracks, e.g., first ply failure, due to a high proof load. Our

theoretical predictions derived in the Appendix are based on the assumption that no damage in fatigue strength is produced by proof tests. The fact that excellent correlation exists between experimental results and theoretical predictions indicates that damage is negligible for the proof load r_0 up to 90% of the mean ultimate strength.

In fact, the ideal experimental testing condition for the verification of damage in fatigue strength is by use of cyclic loading that is below the first ply failure stress. It should be noticed, however, that the endurance limit for Gr/E$\pi/4$ laminates is above the first ply failure stress. As a result, cyclic loading below the first ply failure stress is of no concern in design.

SECTION IV

Conclusion and Recommendation

A theoretical fatigue residual strength degradation model has been derived based on the assumption that the residual strength decreases monotonically. Such a model is applicable to unnotched laminates as demonstrated herein, as well as adhesive or mechanically fastened joints (Refs. 6-8). It is expected to be applicable to laminates with matrix dominated failure mode. This type of residual strength degradation model can also be extended to the case where the loads are random in nature, resembling the real service loads.

For notched laminates, it has been observed that the residual strength increases initially and then decreases eventually. Under such a situation, it is recommended that different theoretical model should be developed. The model should be such that it approaches the strength degradation model described herein as the notched size approaches zero in a limiting case.

We have demonstrated experimentally the validity of both the derived residual strength degradation model and the theory of periodic proof tests using a rather small sample size. The reason for using such a small sample size is due to the limitation that we have only one 2' x 2' panel with square end taps (Fig. 1) available. The panel is cut into 70 specimens. According to our experience, end tap failure can be avoided by using equare end taps. All the data reported herein are not tap failure. It is recommended that large sample size should be used later in order to reduce sampling fluctuation due to the statistical nature of the fatigue behavior of composite laminates.

We have specimens from one panel, referred to as panel 1, with ramped end tap. Unfortunately, the specimen with this type of end tap results in end tap failure under fatigue loads. Limited amount of fatigue data (end tap failure) and static ultimate strength data have also been generated and they also appear to fit the residual strength degradation model described herein, but with quite different parameter values c, b and K. In other words, the shape of the residual strength degradation is different from data without end tap failure.

It has been observed that there may be aging effect for the strength of composite laminates due to residual stress relaxation. Sould this phenomenon occur, the composite strength would increase with time. It should be mentioned that all the data presented in this paper were generated within a one and a half month period. It is, therefore, reasonable to expect that our data should have minimal aging effect.

TABLE 1. ULTIMATE STRENGTH Gr/E $[0,90, \pm 45]_s$, PANEL 3

	Specimen No.	Ultimate Strength ksi
1	1-3	63.152
2	1-2	66.312
3	5-2	71.900
4	1-1	72.323
5	4-1	72.626
6	5-1	75.050
7	3-2	77.743
8	2-3	78.316
9	2-2	80.052
10	4-2	81.324
11	2-1	81.742
12	3-1	84.154
Average		75.39
Standard Deviation		6.12
Coefficient of Variation		0.08
α		14.78
β		78.1

TABLE II. FATIGUE AND RESIDUAL STRENGTH DATA Gr/E $[0,90,\pm 4S]_s$,

PANEL 3

	Specimen No.	Maximum Stress Level S in % of μ	No. of Cycles to Fracture	Residual** Strength
1	1-5	85	1,650	64.012
2	2-4	85	1,950	64.012
3	5-7	85	1,320	64.012
4	5-13	80	2,050	60.246
5	3-4	75	50,980	56.481
6	5-8	75	6,480	56,481
7	4-4	70	155,000	52.716
8	4-3	70	228,500	52.716
9	3-5	70	88,000	52.716
10	1-8	70	117,580	52.716
11	1-12	70	228,700	52.716
12	4-9	70	221,200	52.716
13	2-9	70	310,000	52.716
14	5-6	70	18,790	52.716
15	5-11	70	3,840	52.716
16	4-6	67	161,000	50.456
17	1-11	67	110,000	50.456
18	3-10	65	523,500	48,950
19	2-7	65	863,200	48.950
20	3-8	62	1,346,300	46.691
21	2-12	65	1,007,000*	60.201
22	1-7	72.5	30,000*	61.455
23	1-14	72.5	30,000*	64.544
24	3-13	72.5	30,000*	62.4
25	5-9	72.5	30,000*	61.62

μ = 75.39 ksi = mean ultimate strength.
* = Specimen does not fracture in fatigue.
** = The residual strength at fatigue fracture is equal to S.

TABLE III. RESIDUAL STRENGTH AFTER 26,000 FATIGUE CYCLES AT S = 0.7μ

	Specimen No.	Residual Strength in ksi
1	1-15	66.3
2	1-9	69.1
3	2-8	84.1
4	3-7	78.0
5	4-12	74.3
6	4-13	73.7
7	5-10	62.2
8	5-12	* Fatigue fracture at 17,540 cycles

TABLE IV. RESIDUAL STRENGTH AFTER TWO SERVICE INTERVALS;

$r_0 = 0.9\mu = 67.8$ ksi

	Specimen No.	Residual Strength (ksi)
1	2-10	79.68
2	3-9	63.61
3	4-7	80.17
4	1-10	77.24
5	2-11	75.44
6	3-12	82.03
7	4-10	80.98
8	1-16	71.24
9	2-13	76.31
10	2-14	67.28
11	1-13	69.71
12	1-17	66.10*
13	3-3	62.10*
14	5-14	63.59*

* Specimen fails under the second proof test

TABLE V. RESIDUAL STRENGTH AFTER TWO SERVICE INTERVALS;

$r_0 = 0.85\mu = 64.1$ ksi

	Specimen No.	Residual Strength (ksi)
1	1-4	58.34
2	4-11	67.20
3	2-14	69.74
4	1-6	73.15
5	2-5	75.43
6	3-6	76.02
7	2-6	78.00
8	3-11	80.05
9	5-3	57.73*
10	5-5	62.74*
11	4-5	63.80*

* Specimen fails under the second proof test

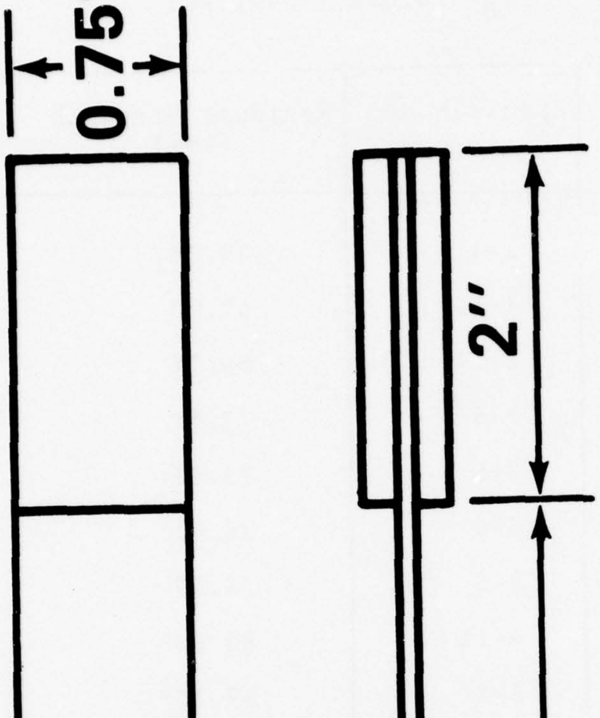

FIG. 1.: SPECIMEN GEOMETRY

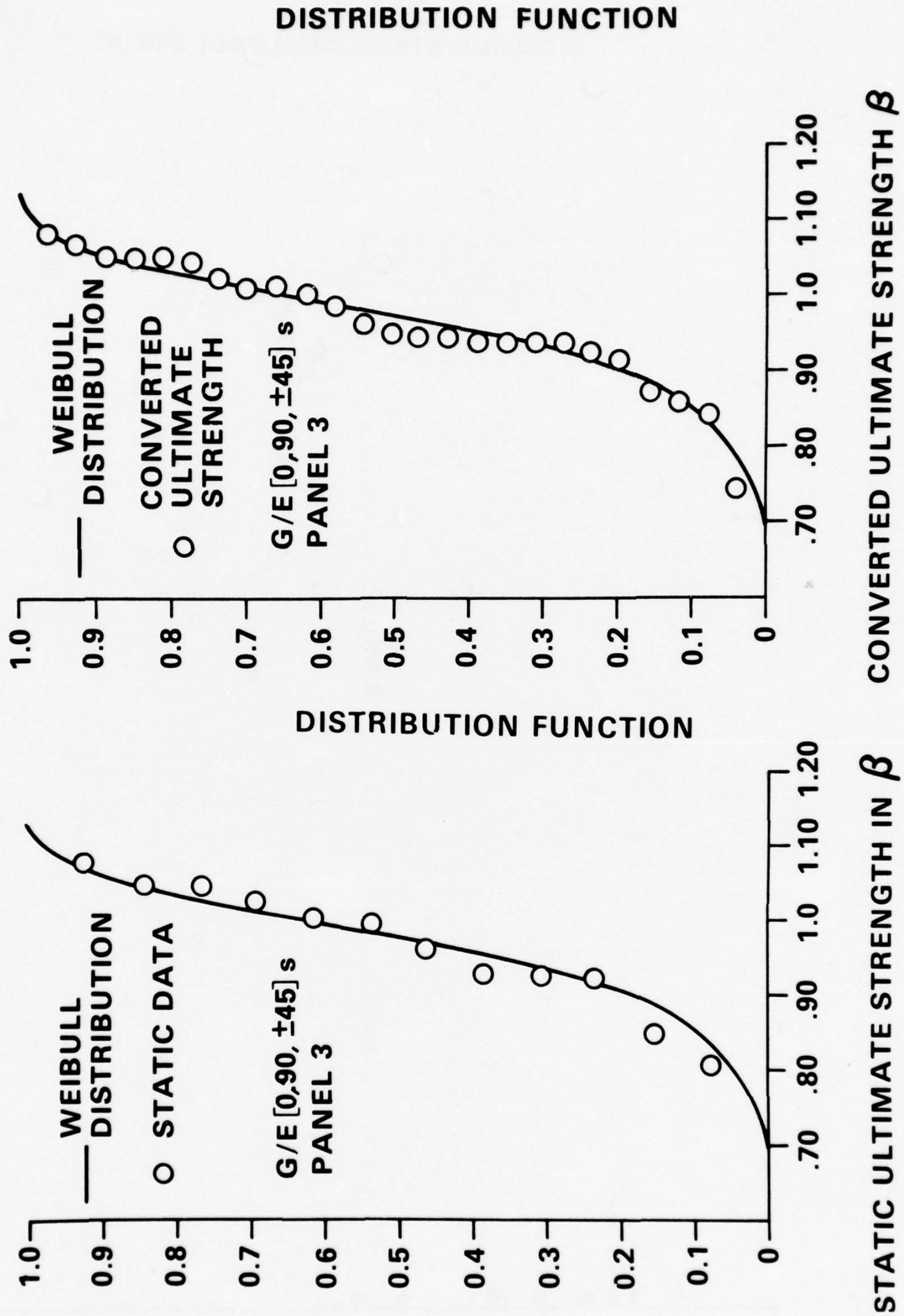

FIG. 2.: DISTRIBUTION OF ULTIMATE STRENGTH; (A) ULTIMATE STRENGTH, (B) CONVERTED ULTIMATE STRENGTH

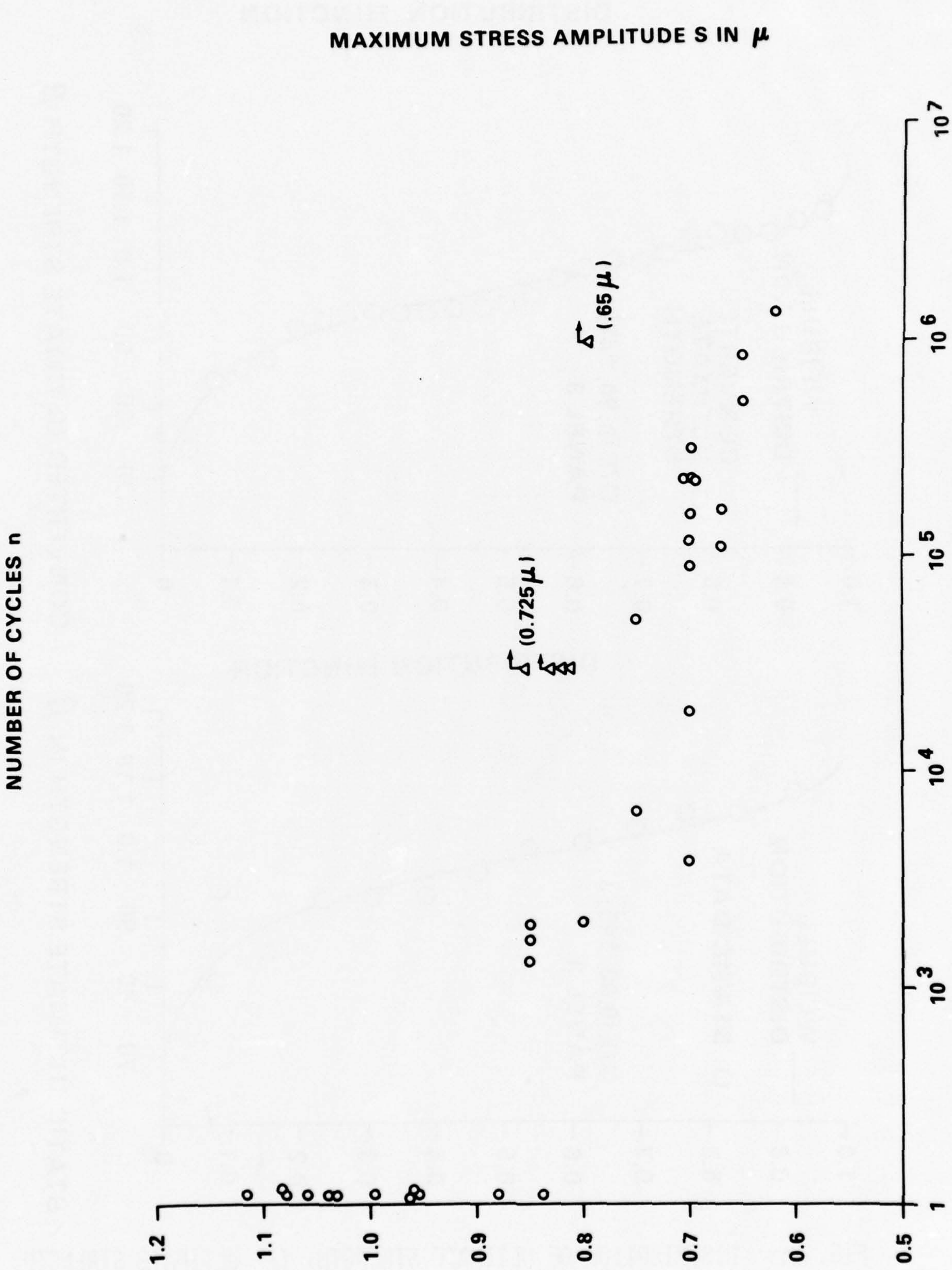

FIG. 3.: ULTIMATE STRENGTH AND FATIGUE DATA

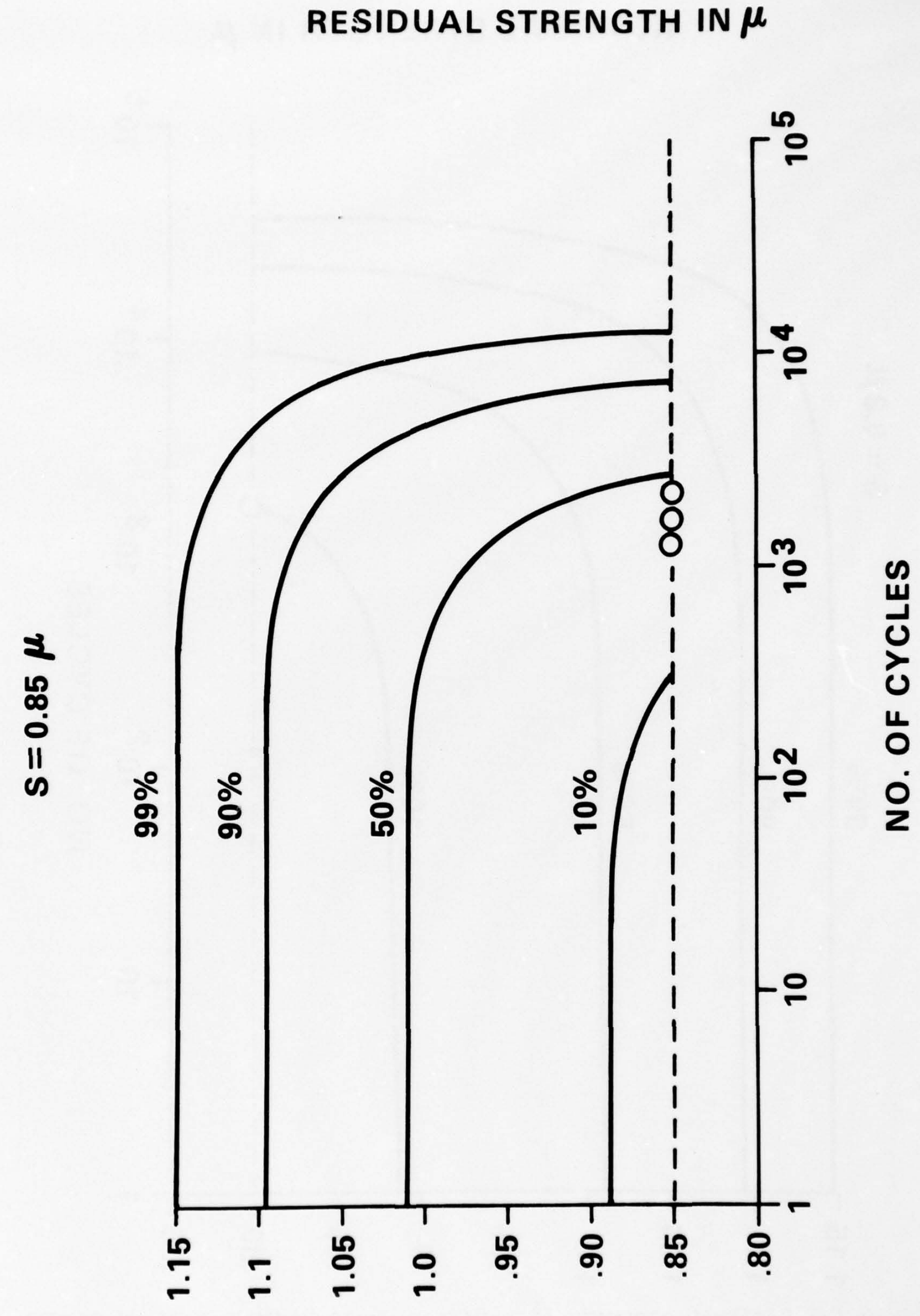

FIG 4.(A) RESIDUAL STRENGTH DEGRADATION UNDER MAXIMUM FATIGUE STRESS LEVEL S = 0.85μ

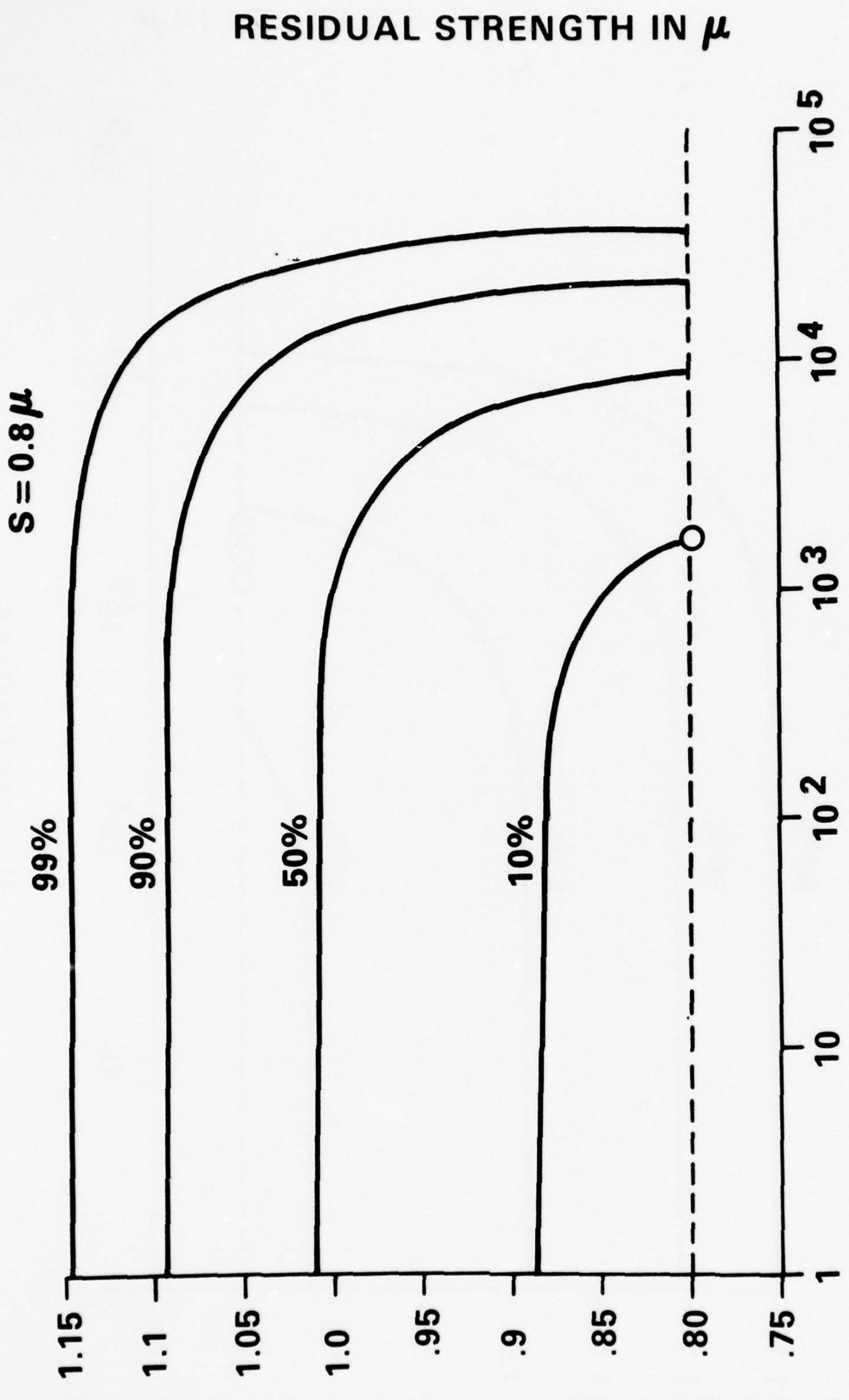

FIG. 4.(B) RESIDUAL STRENGTH DEGRADATION UNDER MAXIMUM FATIGUE STRESS LEVEL S = 0.8μ

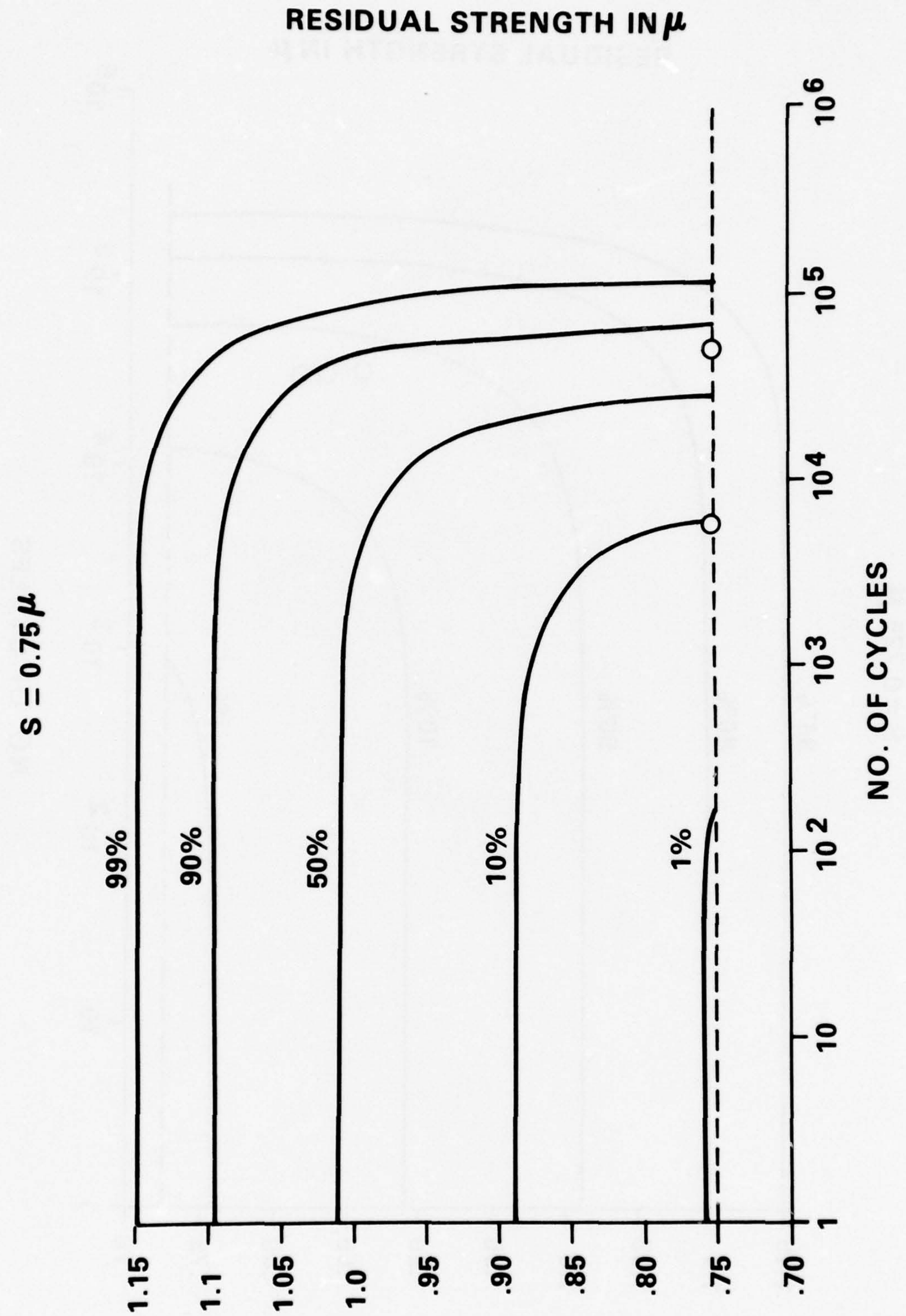

FIG. 4.(c) RESIDUAL STRENGTH DEGRADATION UNDER MAXIMUM FATIGUE STRESS LEVEL S = 0.75μ

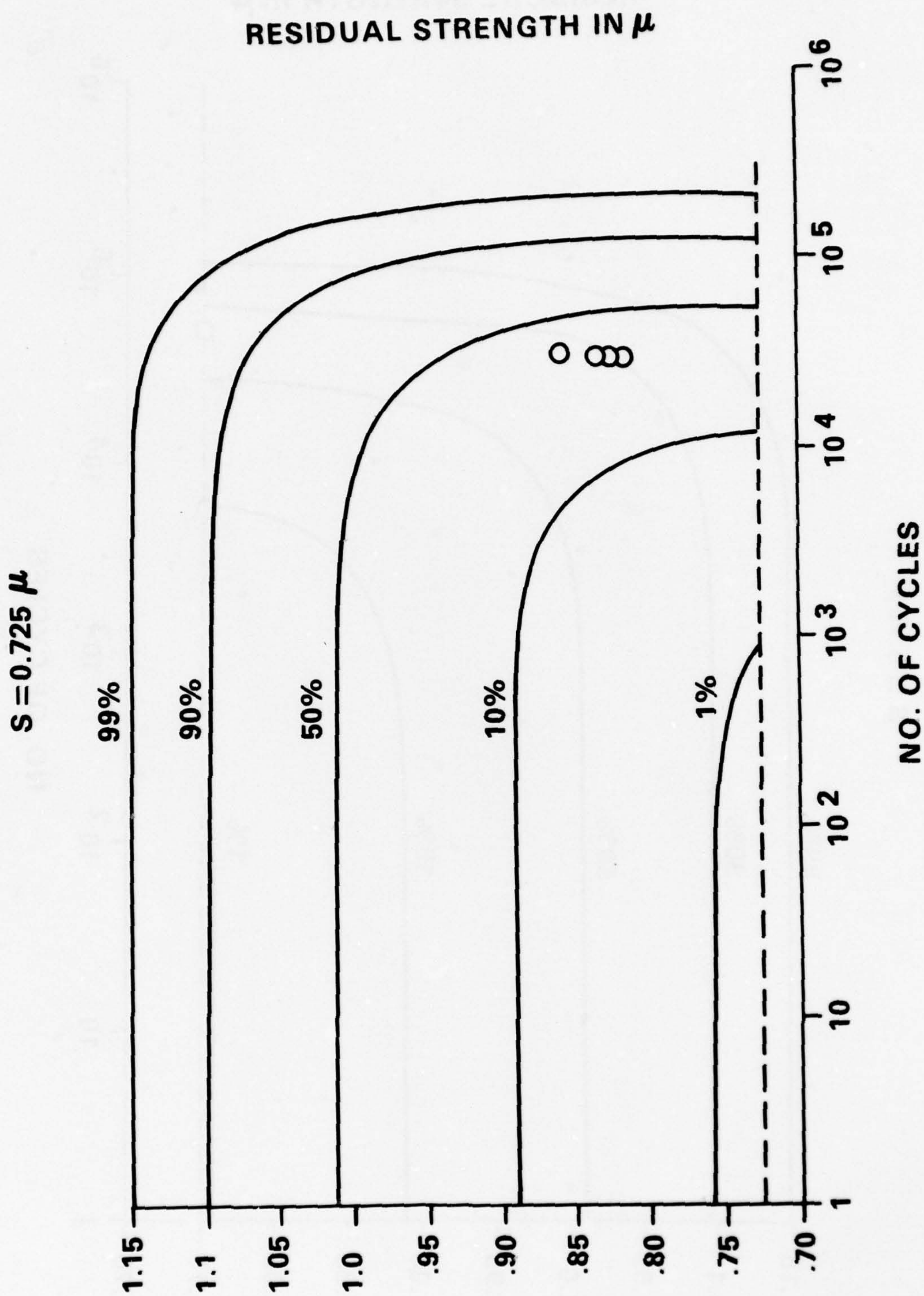

FIG. 4.(D) RESIDUAL STRENGTH DEGRADATION UNDER MAXIMUM FATIGUE STRESS LEVEL S = 0.725μ

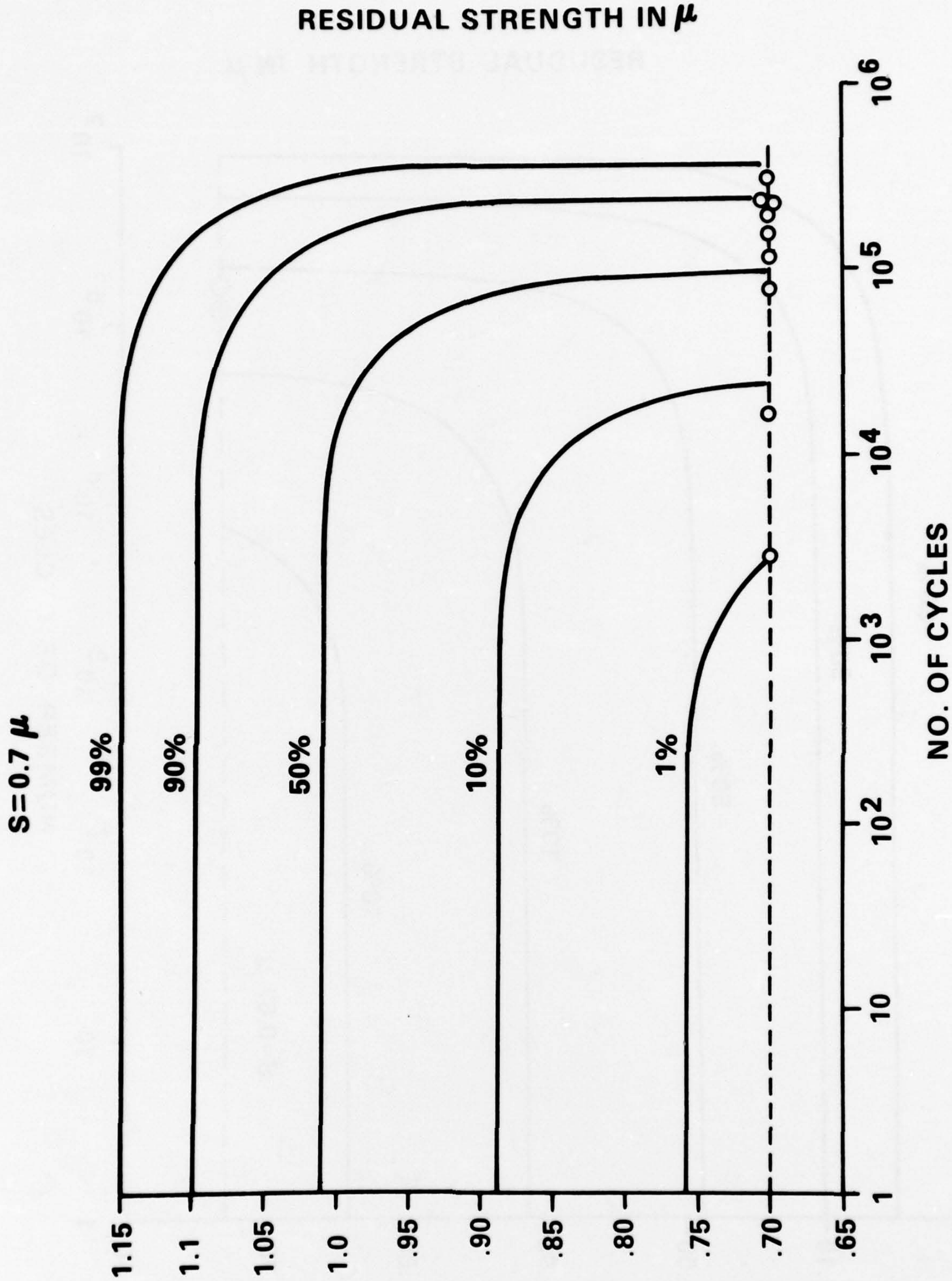

FIG. 4.(E) RESIDUAL STRENGTH DEGRADATION UNDER MAXIMUM FATIGUE STRESS LEVEL S = 0.7μ

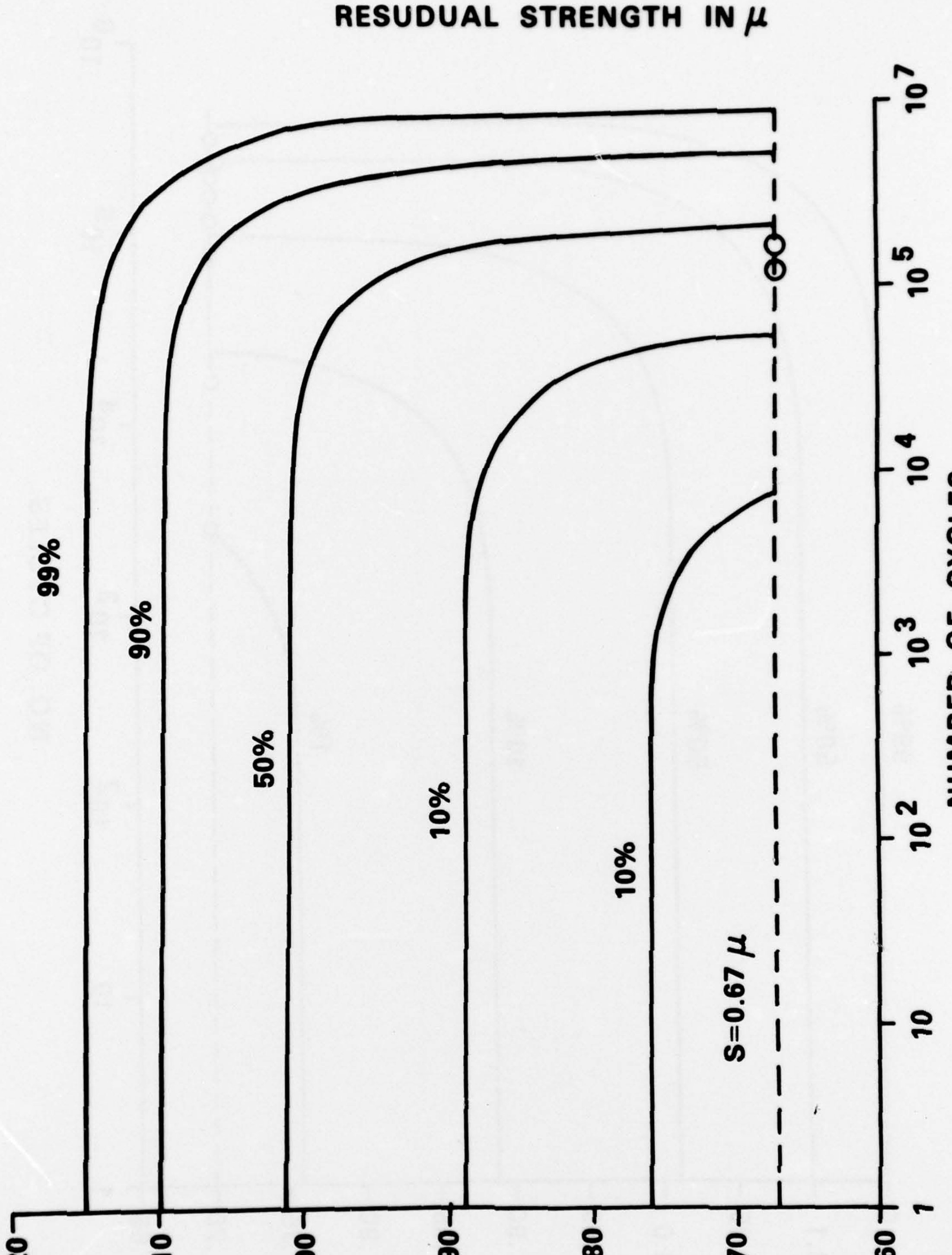

FIG. 4.(F) RESIDUAL STRENGTH DEGRADATION UNDER MAXIMUM FATIGUE STRESS LEVEL S = 0.67μ

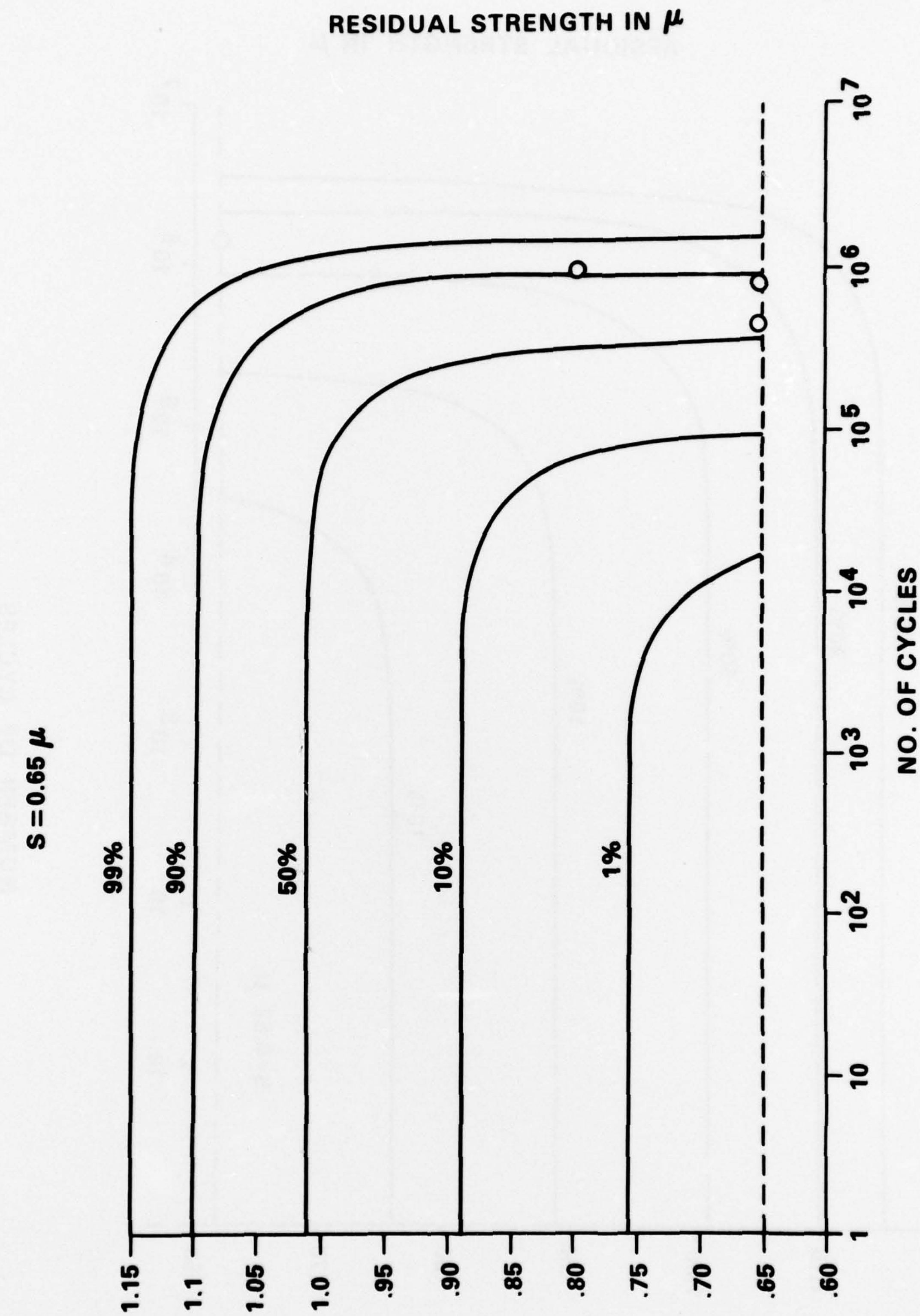

FIG. 4.(G) RESIDUAL STRENGTH DEGRADATION UNDER MAXIMUM FATIGUE STRESS LEVEL S = 0.65μ

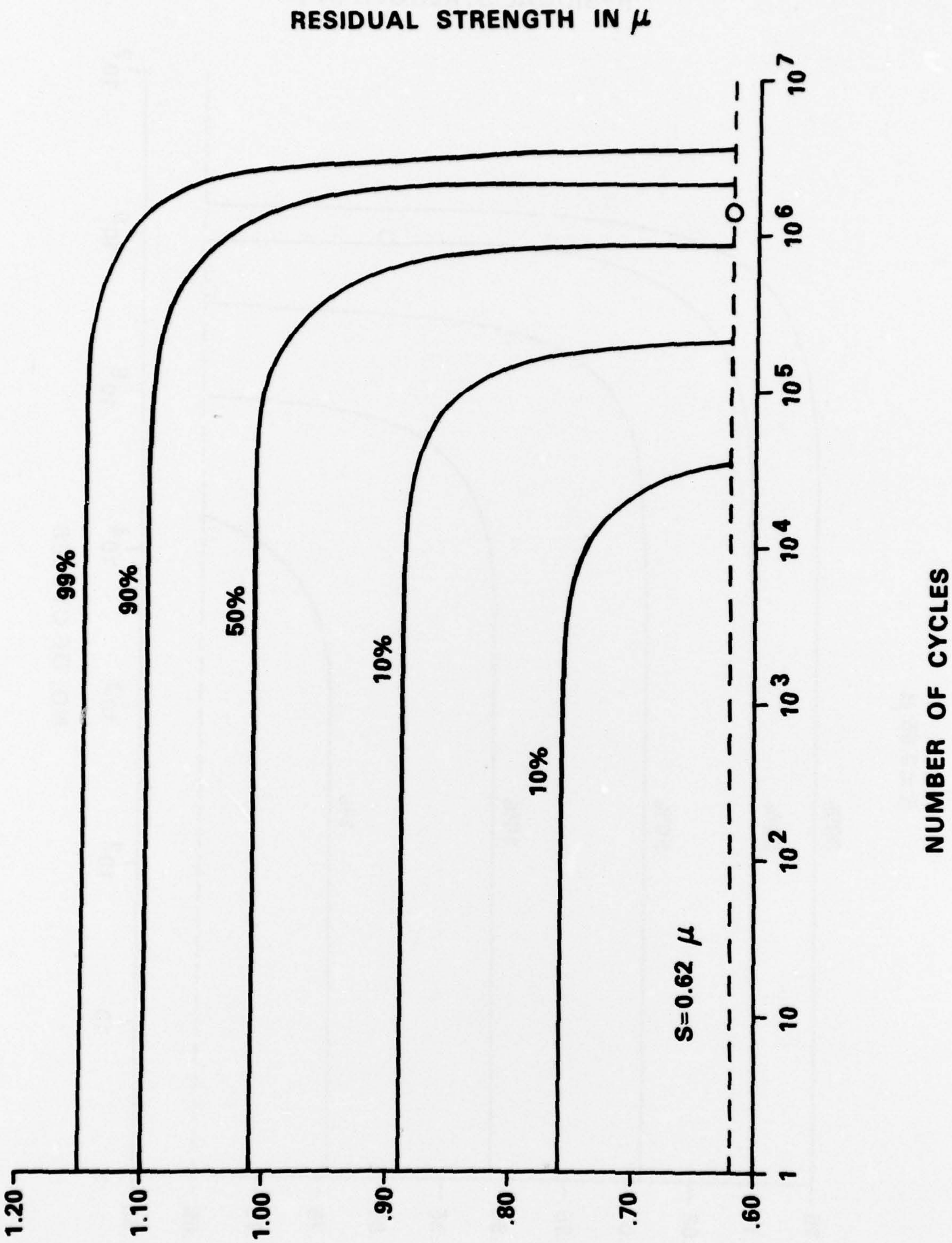

FIG. 4.(H) RESIDUAL STRENGTH DEGRADATION UNDER MAXIMUM FATIGUE STRESS LEVEL S = 0.62μ

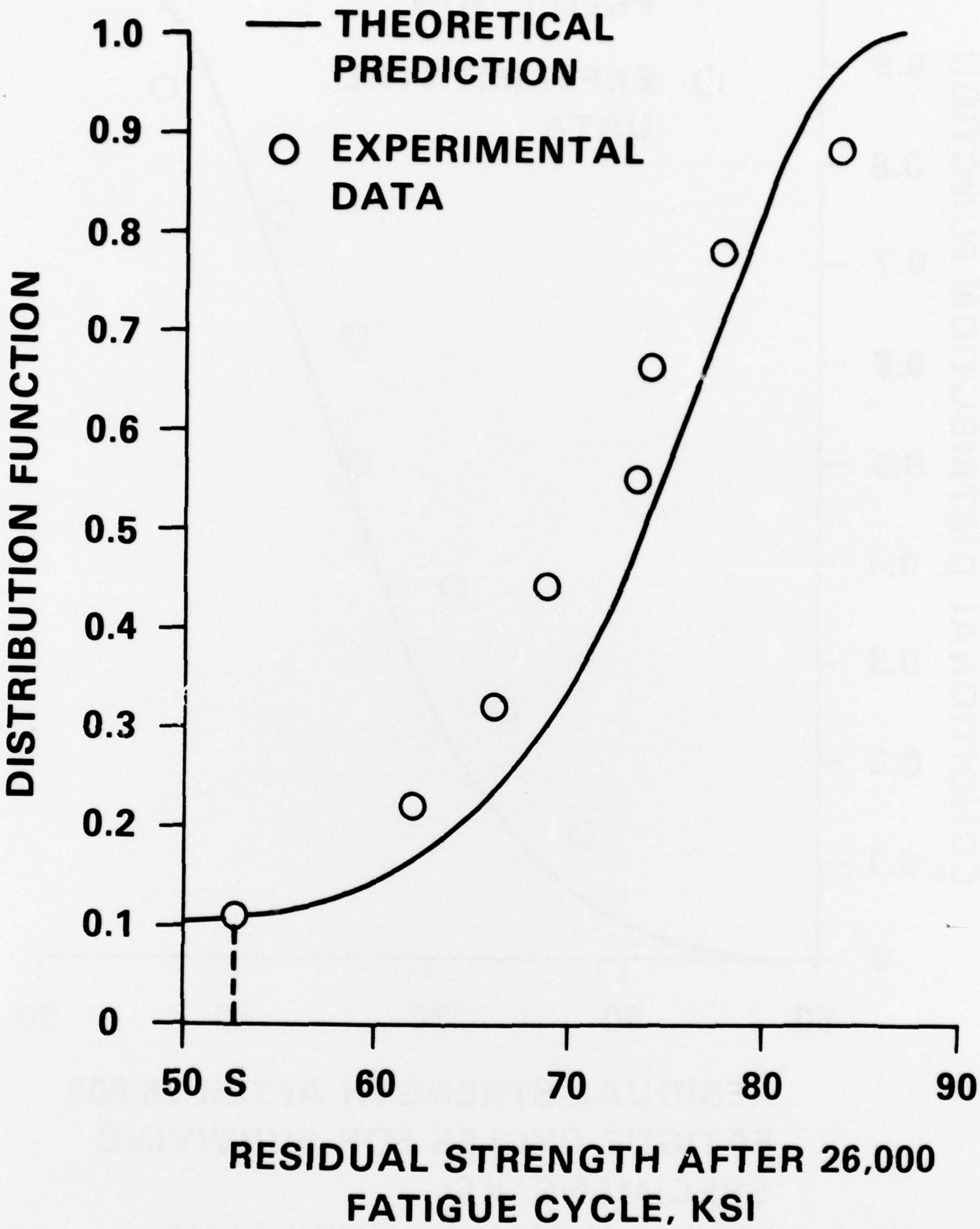

FIG. 5.: STATISTICAL DISTRIBUTION OF RESIDUAL STRENGTH; (A) UNCONDITIONAL DISTRIBUTION

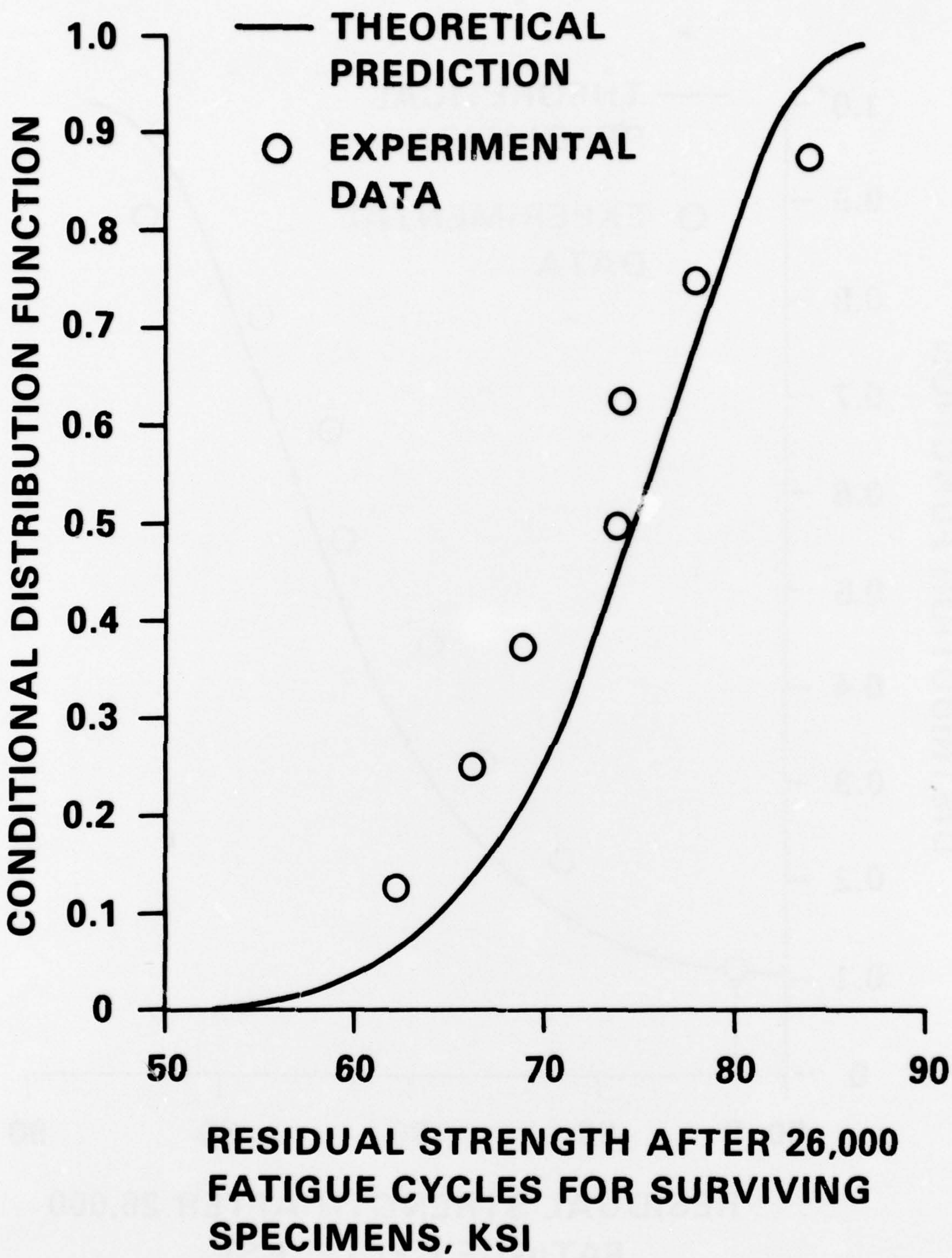

FIG. 5.: STATISTICAL DISTRIBUTION OF RESIDUAL STRENGTH:
(B) CONDITIONAL DISTRIBUTION FOR SURVIVING SPECIMENS

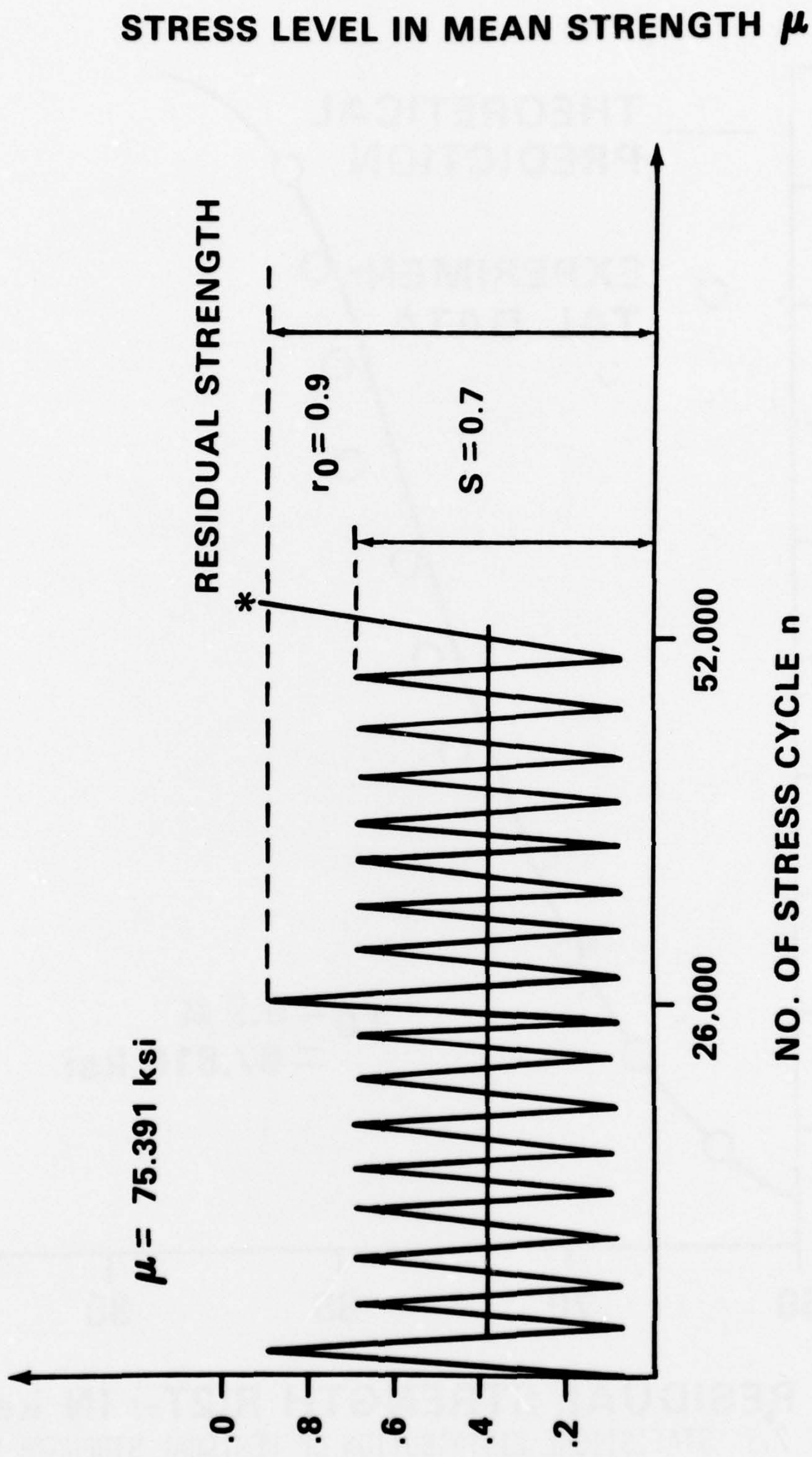

FIG. 6.: LOADING HISTORY

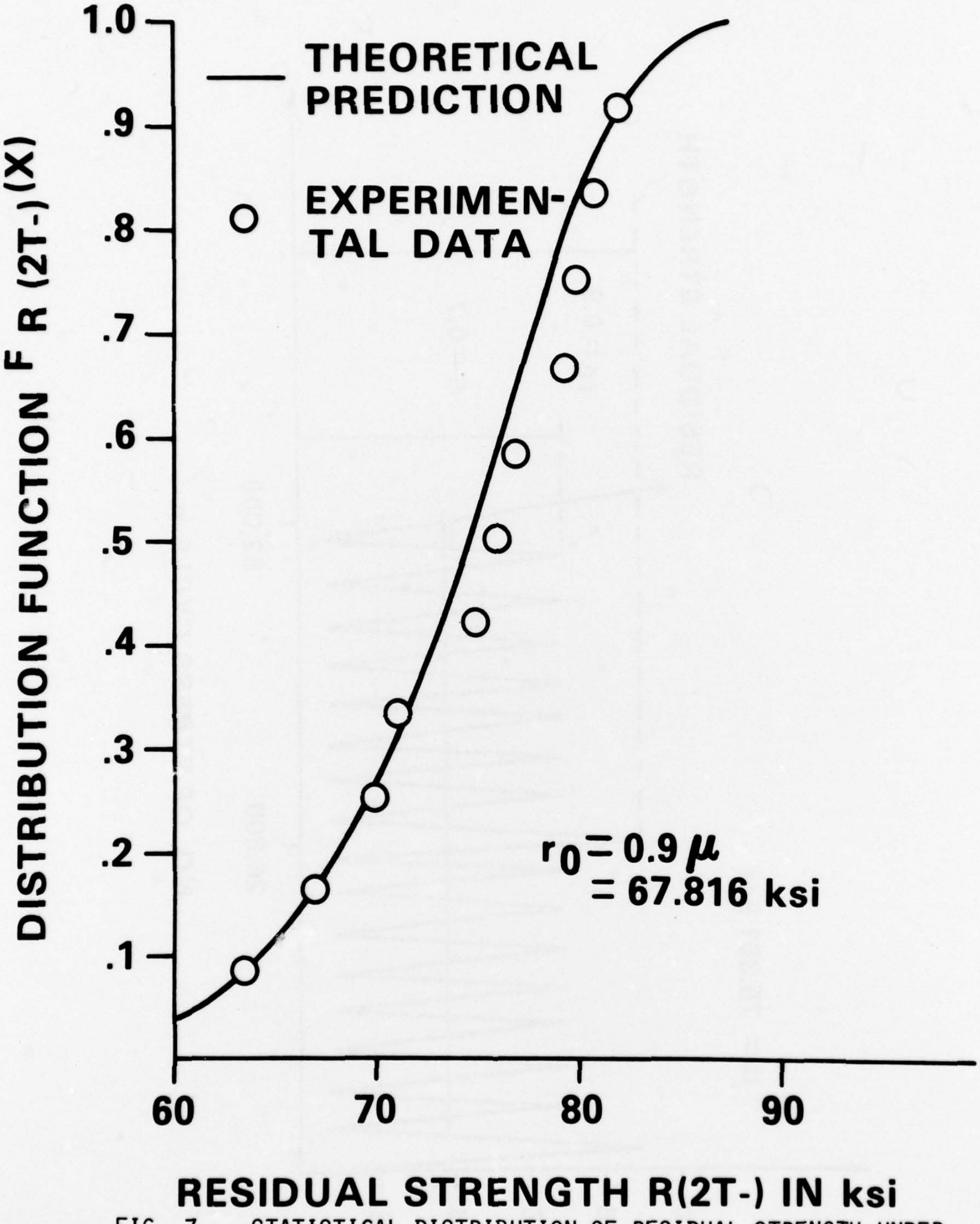

FIG. 7.: STATISTICAL DISTRIBUTION OF RESIDUAL STRENGTH UNDER PERIODIC PROOF TESTS AND FATIGUE LOADS

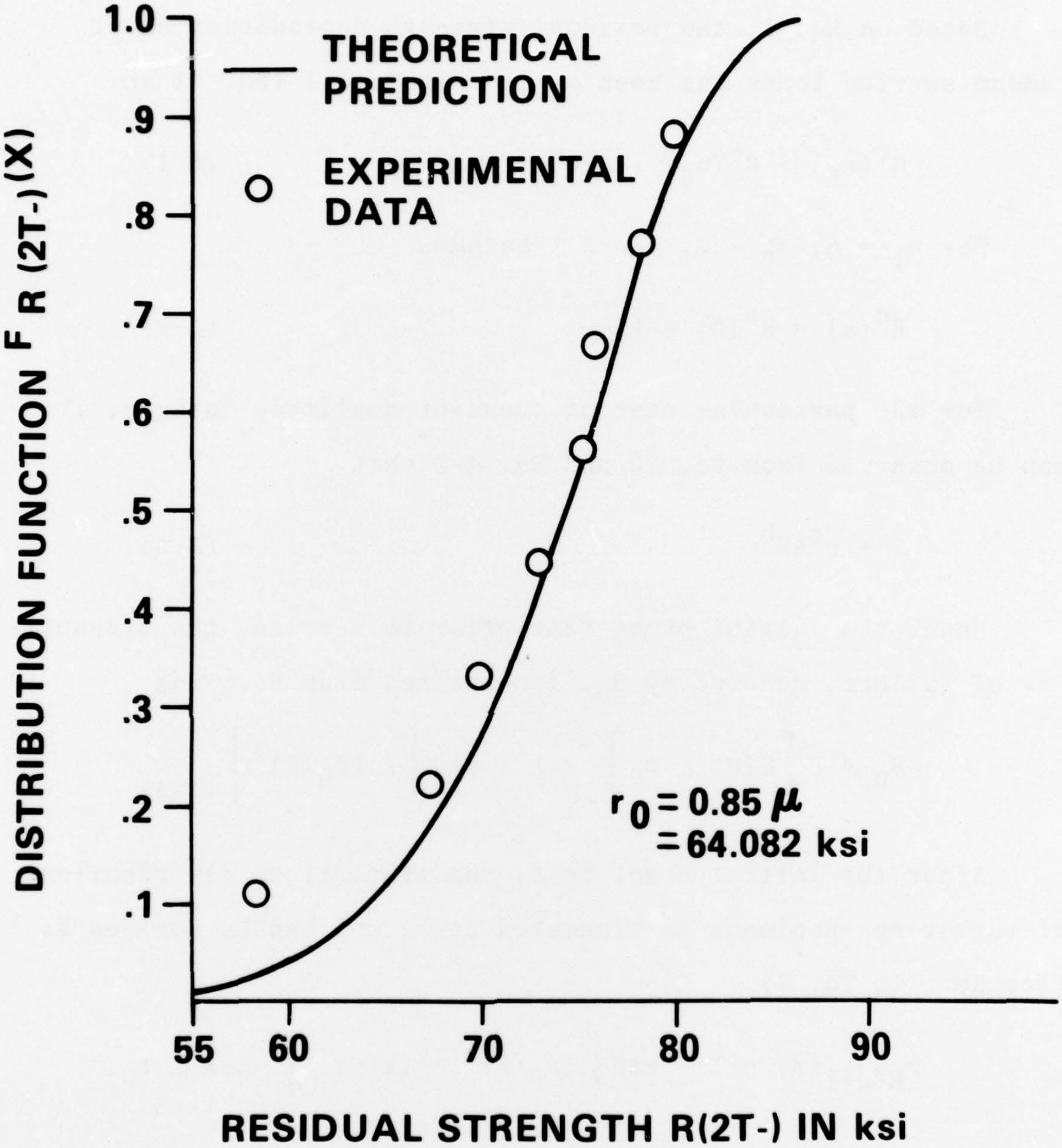

FIG. 8.: STATISTICAL DISTRIBUTION OF RESIDUAL STRENGTH UNDER PERIODIC PROOF TESTS AND FATIGUE LOADS

APPENDIX

PROBABILITY OF FAILURE UNDER PROOF TESTS

Based on Eq. 1, the residual strength degradation under ramdom service loads has been derived in Ref. 1 (Eq. 4) as

$$R^c(n_1) = R^c(n_0) - \Phi(n_1 - n_0) \tag{A-1}$$

For $n_1 = n$, $n_0 = 0$, Eq. A-1 becomes

$$R^c(n) = R^c(0) - \Phi n \tag{A-2}$$

For the particular case of constant amplitude fatigue, it can be observed from Eq. 20 and Eq. A-2 that

$$\Phi = \beta^c K S^b \tag{A-3}$$

Under the initial proof test prior to service, the probability of failure, denoted by B_0, is obtained from Eq. 5 as

$$B_0 = P\left[R(0) \le r_0\right] = 1 - \exp\left\{-(r_0/\beta)^\alpha\right\} \tag{A-4}$$

After the initial proof test, the statistical distribution of surviving specimens is truncated at r_0 and can be derived as * (see Ref. 1, Eq. 2),

$$F_{R(0+)}(x) = 1 - \exp\left\{(r_0/\beta)^\alpha - (x/\beta)^\alpha\right\} \quad ; x \ge r_0 \tag{A-5}$$

---

*In Ref. 1, the ultimate strength of virgin specimen is denoted by R_0, while R(0) represents the ultimate strength of specimens passing the initial proof test. In the present paper, R(0) and R(0+) are, respectively, identical to R_0 and R(0) in Ref. 1.

With the assumption of negligible failure probability in service, the statistical distribution of the residual strength after the second service interval has been derived as (see Ref. 1, Eq. I-7),

$$R_{R(2T^-)}(x) = 1 - \exp\left\{\left(\frac{r_0^c + \Phi T}{\beta^c}\right)^{\alpha/c} - \left(\frac{x^c + 2\Phi T}{\beta^c}\right)^{\alpha/c}\right\} \tag{A-6}$$

in which T is the number of cycles in one service interval (26,000 cycles) and Φ is given by Eq. A-3.

The probability of failure under the second proof test is given by (Ref. 1, Eq. 1-5),

$$1 - B_1^* = 1 - \exp\left\{(r_0/\beta)^\alpha - \left[(r_0^c + \Phi T)/\beta^c\right]^{\alpha/c}\right\} \tag{A-7}$$

Equations given above are applicable to the case where $r_0 = 0.9\mu$, i.e., data in Table IV, since the probability of failure in each service interval is zero. For the case where $r_0 = 0.85\mu$, the probability of failure in each service interval is not negligible, because the specimen passing the proof test can be guarranteed to survive at least 14,000 cycles only. Consequently, the theoretical predictions for the data presented in Table V have to be rederived as follows;

Let r_1, r_2 and r_3 be the static ultimate strength above which the specimen can survive, respectively, the first service interval (26,000 cycles), the second proof test, and second service interval.

r_1 is computed by setting $R(0) = r_1$ and $R(n) = S = 0.7\mu$ in equation 20 for n = 26,000

$$r_1 = \left[S^c + \beta^c K S^b n\right]^{1/c} = 67.684 \text{ ksi} \tag{A-8}$$

r_2 is computed by setting $R(0) = r_2$ and $R(n) = r_0 = 0.85\mu$ in Eq. 20 for n = 26,000 and $S = 0.7\mu$

$$r_2 = \left[r_0^c + \beta^c K S^b n\right]^{1/c} = 70.21 \text{ ksi} \tag{A-9}$$

Finally, r_3 is computed by setting $R(0) = r_3$, and $R(n_1) = S = 0.7\mu$ in Eq. 20 for n_1 = 52,000

$$r_3 = \left[S^c + \beta^c K S^b n_1\right]^{1/c} = 71.93 \text{ ksi} \tag{A-10}$$

The probability of failure in the first service interval, denoted by P_1, is the percentage of specimens having ultimate strength between r_0 and r_1, i.e.,

$$P_1 = F_{R(0+)}(r_1) = 1 - \exp\left\{(r_0/\beta)^\alpha - (r_1/\beta)^\alpha\right\} = 0.064 \tag{A-11}$$

The probability of failure under the second proof test, denoted by B_1, is the percentage of specimens having ultimate strength between r_1 and r_2, i.e.,

$$B_1 = F_{R(0+)}(r_2) - F_{R(0+)}(r_1) = 0.078 \tag{A-12}$$

in which $F_{R(0+)}$ is given by Eq. A-5.

Similarily, the probability of failure in the second service interval, denoted by P_2 is

$$P_2 = F_{R(0+)}(r_3) - F_{R(0+)}(r_2) = 0.073 \tag{A-13}$$

As a result, all the specimens with ultimate strength less than r_3 will not survive up to the end of the second service interval. The statistical distribution of the ultimate strength, $\tilde{R}(0)$, of the surviving specimens is therefore truncated at r_3,

$$\begin{aligned} F_{\tilde{R}(0)}(x) &= 1 - P\left[R(0+) > x \mid R(0+) > r_3 \right] \\ &= 1 - \left\{ P\left[R(0+) > x \right] / P\left[R(0+) > r_3 \right] \right\}; \; x \geq r_3 \end{aligned} \tag{A-14}$$

Substitution of Eq. A-5 into Eq. A-14 leads to the following:

$$F_{\tilde{R}(0)}(x) = 1 - \exp\left\{ (r_3/\beta)^{\alpha} - (x/\beta)^{\alpha} \right\}; x \geq r_3 \tag{A-15}$$

The ultimate strength, $\tilde{R}(0)$, of the surviving specimens is subjected to degradation following Eq. 20, i.e.,

$$R^c(n) = \tilde{R}^c(0) - \beta^c K S^b n \tag{A-16}$$

in which n = 52,000 and R(n) is the residual strength at the end of the second service interval. The statistical distribution for R(n) is obtained from that of $\tilde{R}(0)$ given by Eq. A-15 through the transformation of Eq. A-16 as follows;

$$F_{R(n)}(x) = 1 - \exp\left\{ \left(\frac{r_3}{\beta} \right)^{\alpha} - \left(\frac{x^c + \beta^c K S^b n}{\beta^c} \right)^{\alpha/c} \right\} \tag{A-17}$$

in which r_3 = 71,93 ksi and n = 52,000.

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